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Further, if Venus be the disturbed planet and the Earth the disturbing planet, then

$\alpha = 0{\cdot}723\ 331\ 5 \quad = a$
$\mathrm{a} = 1. \qquad\qquad = a'$
$\log \sigma = 8{\cdot}471\ 368$
$\omega = 234^\circ\ 7'\ 22''{\cdot}2 \qquad \epsilon = {\cdot}003\ 421\ 70$
$\omega' = 205^\circ\ 1'\ 29''{\cdot}1 \qquad \epsilon' = {\cdot}008\ 386\ 14$
(see Newcomb, *A.P.*, iii. p. 406, 459)

and

$$\mathrm{D}\alpha^m = m\alpha^m .$$

The principal term becomes

$$-\frac{\epsilon^3\epsilon'}{2} \cdot 96 \cdot \sigma^2\alpha^5 \cdot \frac{\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\frac{9}{2}}{1\cdot 2\cdot 3\cdot 4}$$

which is less than 10^{-11}, and has still to be multiplied by the mass of the Earth.

Note on Major MacMahon's paper "On the Determination of the apparent Diameter of a fixed Star." By A. S. Eddington, M.A., M.Sc.

In Major MacMahon's paper (p. 126 *ante*), the fading of the light of a star at the instant of disappearance behind the Moon was treated purely as a problem of geometrical optics. When account is taken of the diffraction of the star's light by the limb of the Moon, his results will be found to require some modification. The argument given in this note appears to show that, judged by the rate of diminution of its light at occultation, every star will seem to have an angular diameter of about 0″·008, due to this diffraction.

Reducing the problem to its ideally simple form, we consider a point-star S. The Moon is treated as a screen M M′ with a straight edge through M perpendicular to the plane of the paper.

Let X E X′ be a screen in the position of the observer, E being the boundary of the geometrical shadow cast by the Moon. Let J be the intensity of illumination of the screen by the star when the Moon is out of the way, the true illumination of the screen will then be as follows:[*] —

Inside the geometrical shadow, the illumination is zero until near the edge; then it slowly increases, reaching the value $\frac{1}{4}$ J at E; outside the geometrical shadow it still increases, until it reaches

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[*] See, for instance, A. Schuster, *Theory of Optics*, p. 94.

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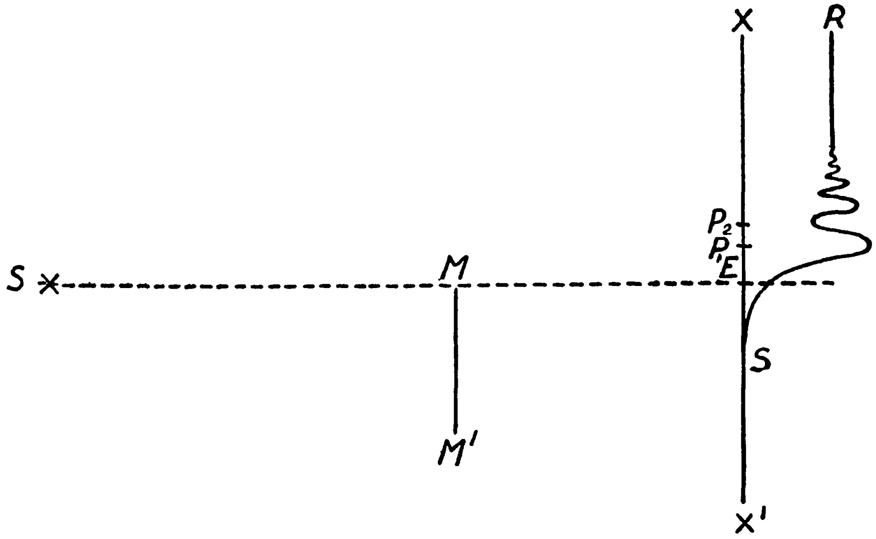
a maximum of 1·37 J at P_1; it then decreases to a minimum of 0·78 J at P_2; beyond these are a series of maxima and minima, becoming rapidly less marked, and finally merging into a uniform illumination J. The curve R S shows by its varying distance from X X′ the varying intensity of the illumination along the screen. The most important part of the change is between P_1 and E, where the illumination changes from 1·37 J to 0·25 J. Thus if we consider the observer at the time of occultation to be carried gradually into the shadow, the time taken to pass from the configuration S M P_1 to the configuration S M E will be an approximate measure of the time taken for the star to disappear, and the angle S P_1 M will be a measure of the spurious diameter of the star as deduced from its behaviour at occultation. Actually about $\frac{4}{3}$ S P_1 M should be taken as the equivalent diameter, in order to allow for the fact that the illumination at E is still rather more than a sixth of that at P_1.



Now P_1 is given by the condition

$$SM + MP_1 - SP_1 = \tfrac{3}{8}\lambda \quad \text{approximately}$$

where λ is the wave-length of light.

Remembering that S M is almost infinitely greater than M E the condition reduces to

$$MP_1 - ME = \tfrac{3}{8}\lambda.$$

Therefore
$$1 - \cos P_1ME = \tfrac{3}{8}\frac{\lambda}{MP_1}$$

$$\therefore \quad \angle P_1ME = \sqrt{\tfrac{3}{4}\frac{\lambda}{MP_1}}.$$

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Filling in the values

$$\lambda \text{ (photographic)} = 4{\cdot}3 \times 10^{-5} \text{ cms.}$$

$$MP_1 = 380000 \text{ kilometres.}$$

We find

$$\angle SP_1M = \angle P_1ME = 0''{\cdot}006.$$

Further, $P_1E = 36$ feet.

The spurious occultation diameter due to the diffraction of light at the Moon's limb is thus about 0″·008.

In laboratory diffraction problems it is usually rather difficult to determine what is the effect of using a telescope for direct observation of the diffraction pattern instead of studying the illumination of a screen, because the aperture of the telescope occupies a space which would be covered by a great many diffraction bands. In the present case, however, the difficulty does not arise: P_1E being 36 feet, the aperture of the telescope is very small compared with the width of a diffraction band; and consequently, when the telescope is pointed directly to the star, the change of phase across the aperture (the crinkling of the wave-front) is negligible. The telescope therefore receives plane waves from the star, the intensity varying according to the law we have already considered. Thus the star is seen in its normal position, its brightness varying in the same way as the illumination on a hypothetical screen, coinciding with the telescope. Of course the star will have the ordinary spurious apparent diameter, depending on the aperture of the telescope, very much greater than, and independent of, the spurious occultation diameter which we have been considering. It may seem surprising that the only effect of diffraction is to alter the apparent brightness of the star, for in the laboratory the effect is to border the obstacle with bright and dark bands. These are not seen in the astronomical problem, because, being of the order ″·006 in width, the resolving power of the telescope is far too small to distinguish them.

It is hardly necessary to consider in detail the other respects in which the actual problem differs from one of diffraction by a straight edge. The fact that the Moon is a globe, and not a flat disk, may affect the results, and so also may the irregularities of its contour. (The first Fresnel zone at the Moon's surface is about 42 feet in radius.) The fact that the precise effect of diffraction cannot be certainly estimated would rather add to the dificulties of applying the proposed method. But whether this possibility is serious or not, the normal diffraction diameter of 0″·008 is sufficiently great to mask the true diameter in all but very exceptional cases.

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Some Remarks on Lambert's Theorem. By H. C. Plummer, M.A.

1. It is a curious fact that the proofs of a theorem so celebrated as Lambert's, as they appear in at least the majority of text-books, are very unsatisfactory in the treatment given to the ambiguities of sign which must occur in the statement of the theorem. In most of the cases which arise in practice the arc of the orbit is comparatively short, and there is really no question about the signs to be chosen. This, coupled with the fact that it was commonly supposed that the form of the theorem was in general unsuitable for use in the determination of orbits, has apparently caused a certain carelessness in the demonstration. Recent work has, however, tended to place the value of the theorem more clearly in evidence.* For the case of elliptic motion the essence of the matter in regard to the distinction in the ambiguous signs has been given by Callandreau.† For hyperbolic motion under attraction to a centre a simple proof was given by Adams ‡ in terms of hyperbolic functions, but the question of signs was ignored by him altogether. The case of motion under a repulsive force along the convex branch of an hyperbola does not seem to have been discussed at all, though the motion of matter in a comet's tail may give this case a real practical importance. It may therefore be useful to give here some examination to the methods of proof in the several cases, with such remarks as may be suggested incidentally.

2. The case of elliptic motion will be discussed first. Let E_1, E_2 be the eccentric anomalies of the starting and end points respectively, and let $E_2 > E_1$, both angles being less than 360°. In the first instance we suppose that the motion is in the same sense as the anomaly increases. It is easy to prove that, μ being the force at unit distance,

$$\mu^{\frac{1}{2}} a^{-\frac{3}{2}} t = \epsilon - \delta - (\sin \epsilon - \sin \delta)$$

where

$$\sin^2 \tfrac{1}{2}\epsilon = (r_1 + r_2 + c)/4a \equiv m_1{}^2, \quad \sin^2 \tfrac{1}{2}\delta = (r_1 + r_2 - c)/4a \equiv m_2{}^2$$

The relations of ϵ and δ to the eccentric anomalies are given by

$$\epsilon - \delta = E_2 - E_1, \quad \cos \tfrac{1}{2}(\epsilon + \delta) = e \cos \tfrac{1}{2}(E_2 + E_1)$$

The angle $\frac{1}{2}(\epsilon + \delta)$ is merely defined by the latter relation, and may be taken between the limits 0° and 180°; and $\frac{1}{2}(\epsilon - \delta)$ must lie between the same limits. Hence ϵ must lie between 0° and 360°, while δ must lie between ± 180°. It follows that $\sin \frac{1}{2}\epsilon$ and $\cos \frac{1}{2}\delta$ are both positive. If, then, ϵ_1 and δ_1 are the principal values resulting from

$$\sin \tfrac{1}{2}\epsilon = +m_1, \ \sin \tfrac{1}{2}\delta = +m_2$$

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* *Cf. M.N.*, lxiii. p. 147.

† *Annales de l'Obs. de Paris, Mémoires*, vol. xxiii.

‡ *Collected Scientific Papers*, vol. i. p. 411.